CONO-CUNEUS:

OR, THE

SHIPWRIGHT'S CIRCULAR WEDGE.

THAT IS,

A Body resembling in part a *Conus*, in part a *Cuneus*, Geometrically considered.

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IN A

LETTER

TO THE HONOURABLE

Sir ROBERT MORAY, *Knight.*

LONDON:

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TO THE HONOURABLE

Sir Robert Moray, Kt.

SIR,

SInce I came home from London, I have taken some time to consider of those Solids and Lines made by the Sections thereof; proposed to Consideration (to my Lord Brouncker and your self, at your Lodgings, where I was also present) by Mr. Pett, one of His Majesties Commissioners for the Navy, and an excellent Shipwright.

The Bodies proposed to consideration were all of this form. On a plain Base, which was the Quadrant of a Circle, (like that of a Quadrantal Cone or Cylinder) stood an erect Solid, whose Altitude (being arbitrary) was there double to the Radius of that Quadrant; and from every Point of its Perimeter, streight Lines drawn to the Vertex, met there, not in a Point (as is the Apex of a Cone), nor in a parallel Quadrant (as in a Quadrantal Cylinder), but in a streight Line or sharp Edge, like that of a Wedge or Cuneus. On which consideration, I thought fit to give it the name of Cono-Cuneus, as having the Base of a Cone, and the Vertex of a Cuneus.

By the various Sections of this Solid, in several Positions, he did (rightly) conceive, that divers new Lines must arise, in great variety, different from those arising from the Section of a Cone. Some of which he supposed might be of good use in the Building of Ships; in order to which it was, that he proposed them to Consideration

Now because he judged it troublesom (as indeed it would be) first to form such Solids, and then cut them by Plains in such Positions as he desired; he had (for avoiding that trouble) ingeniously contrived this Expedient. He caused divers Boards, of a good solid Wood, to be exactly planed, some of an equal thickness, some meeting in a sharp edge; those of the former, he caused to be glewed together in a parallel Position; those of the latter sort, he caused so to be glewed together, as that their sharp edges met in one common Angle. And having thus formed several Solids, of Boards thus glewed together, he then caused them to be wrought into such a form as that before described: Which being done, he then caused the Glew

to be dissolved in warm Water, whereby the several Boards, falling asunder, did exhibit, in their several faces, the respective Sections of those Solids. And such were those he shewed us; which being put together, made up such Solids; and taken asunder, shewed the several Sections of them.

I do not intend at all to disparage the ingenuity of that Contrivance, which was indeed very handsom, and neatly performed, but do withall suppose, that it would not be unpleasing to your self, or him, to see those Lines described in Plano, which would arise by such Section of the Solid.

That therefore is the work of these Papers, to represent the true nature of such Lines, and the ways to draw them, without the actual Section of a Solid.

Which I have the rather undertaken, because this is a Solid which I do not know that any other have before considered. And because this may be a Pattern; according to which, other Solids of like nature may be in like manner considered if there shall be occasion.

If beside these Sections which he hath already considered, there be any other Sections of this or other the like Solids which he shall conceive useful to his purpose; the same may in like manner be represented (without the actual Section of such Solids) by Lines thus described in a Plain.

But which of them may be most advantageous to his design, I do not pretend to understand so well, nor can with so much certainty affirm; as, that I am,

SIR,

Oxon, Apr. 7. 1662. *Your very humble Servant,*

JOHN WALLIS.

CONO-

CONO-CUNEUS
OR, THE
SHIPWRIGHT'S
CIRCULAR WEDGE.

The Sections of a CONO-CUNEUS.

1. IN a Rectangle CDBA, *Fig.* 1. erect at Right Angles the Quadrant of a Circle CQD; and joining QA, compleat the Rectangled Triangle CQA. Supposing then from every Point of the Quadrantal Arch DQ, to their respective Points in the streight Line BA, in Plains parallel to the Triangle CQA, the streight Lines Sa to be drawn, compleating a Curve Superficies DSQAaB; the Solid thus contained, I call a *Cono-Cuneus*.

2. It differs from a Quadrantal Cone, in this only; That what is here a streight Line AB, is there a single Point; all the Lines drawn from the Points S, meeting there at the Point A.

3. It differs in this from a Wedge, or *Cuneus*; That what is here a Quadrant CQD, is there a Rectangle.

4. It differs in this from a Quadrantal Cylinder; That what is here a streight Line AB, is there a Quadrant, equal and parallel to CQD.

5. This Solid, being cut by Plains in different Positions, will produce, in the Curve Surface DQAB, great variety of Lines. *As for Example:*

6. *First*; If it be cut by RSa, a Plain parallel to the Triangle CQA, the Line Sa is (by construction) a streight Line; and therefore, the Hypothenuse of a Right-angled Triangle SRa.

7. And consequently, this *Cono-Cuneus* is equal to half a Quadrantal Cylinder of the same Base and Altitude: For every of the Triangles SRa in the *Cono-Cuneus*, being half the respective Rectangle in the Cylinder, the whole of That will be equal to the half of This.

8. The Quantities therein I thus design in *Species*.

CS = CD = *A*.
CR = *c*.
RS = *r* = $\sqrt{:A^2 - c^2}$.
Ra = CA = *A*.
SA = $\sqrt{:A^2 + r^2}$ = $\sqrt{:2A^2 - c^2}$.

Fig. 1.

9. These Triangles, (if made by Plains let at equal distances) projected on the Plain CQA to which they are parallel, will appear as in the first Projection, *Fig.* 16. which is thus drawn: Having drawn a Triangle ACQ, like and equal to that in the Solid, and CQD the Quadrant of a Circle, let CD be divided into any number of equal parts at the Points R; from every of which, the Ordinates RS being drawn, take equal thereunto, in the Line CQ, the Lines Cs,

of

or R s; then joining A s, the Triangles s R A or s C A in this Plain, repreſent the like Triangles S R A in the Solid.

10. And if we ſuppoſe the Solid to be continued downward, beyond its Quadrantal Baſe, theſe Triangles muſt be ſo continued alſo: And the like, if we ſuppoſe it to be continued upward, (after a decuſſation in A B) as in oppoſite Cones.

11. The Quantities, in this Projection, I deſign thus, in *Species*.

Fig. 16.

$$\begin{aligned} CQ = CD &= R. \\ CR &= c. \\ CS = RS &= \sqrt{:} R^2 - c^2. \\ AC &= A. \\ sA &= \sqrt{:} A^2 + R^2 - c^2. \end{aligned}$$

12. In Numbers thus; (putting $R = 1$. $A = 2$.)

C R.	C s.	A s.
0.	1.	2.236+
0.125	0.992+	2.233—
0.25	0.968+	2.222+
0.375	0.927—	2.204+
0.5	0.866+	2.179+
0.625	0.781—	2.147—
0.75	0.661+	2.106+
0.875	0.484+	2.058—
1.	0.	2.

13. *Secondly*; If it be cut by E d q, a Plain parallel to the Quadrantal Baſe C D Q, *Fig.* 1. the Curve Line d σ q will be an Ellipſe: For (ſuppoſing this Plain to be cut in ϱ σ by R S a, any of thoſe Triangles parallel to C Q A;) then is, *As* A C *to* A E, or a R *to* a ϱ; *So* C Q *to* E q, and R S *to* ϱ σ. And conſequently (the Ordinates ϱ σ being proportional to R S the Ordinates of a Circle) E d q will be the Quadrant of an Ellipſe, as C D Q is of a Circle.

14. The Quantities I thus deſign in *Species*.

Fig. 1.

$$\begin{aligned} CQ = CD &= R. \\ CR &= c. \\ RS &= s = \sqrt{:} R^2 - c^2. \\ qE &= E. \end{aligned}$$

$$AC \,.\, AE :: CQ\,(=Ed) \,.\, qE :: RS \,.\, \varrho\sigma.$$

That is, $R \,.\, E :: \sqrt{:} R^2 - c^2 \,.\, \frac{E}{R}\sqrt{:} R^2 - c^2$.

And therefore, $\varrho\sigma = \frac{E}{R}\sqrt{:} R^2 - c^2$.

15. Theſe Ellipſes (if cut off by Plains ſet at equal diſtances) projected on the Quadrant C D Q (to which they are parallel) will appear as in the ſecond Projection, *Fig.* 17. which is thus drawn: Having drawn a Quadrant C D Q equal to that in the Solid, let C Q be divided into any number of equal parts at the Points q, and every of the Ordinates R S (parallel thereunto) at the Points σ; through which, if we draw the Ellipſes D σ q, theſe in the Plain will repreſent the like Ellipſes d σ q in the Solid.

16. If the Solid be ſuppoſed to be continued downward below its Quadrantal Baſe C D Q, the parallel Sections will yet be Ellipſes: But with this difference; C D, which is now half the longeſt Diameter, will then be half the ſhorteſt Diameter of the Ellipſe, (ſuch as are thoſe in *Fig.* 17. beyond the Circular Quadrant D Q:) And if the Solid be continued upward, after a decuſſation in A B, the like Ellipſes will occur in the oppoſite Solid as in this.

17. The

17. The Quantities in this Projection I thus design in *Species*.

$$CQ = CD = R.$$
$$CR = c.$$
$$RS = s = \sqrt{:R^2 - c^2}.$$
$$Eq = E.$$
$$CQ \,.\, Eq :: RS \,.\, \varrho\sigma.$$
$$R \,.\, E :: \sqrt{:R^2 - c^2} \,.\, \frac{E}{R}\sqrt{:R^2 - c^2}.$$
$$\varrho\sigma = \frac{E}{R}\sqrt{:R^2 - c^2}. = \frac{E}{R}s.$$

Fig. 17.

18. In Numbers thus; (putting $R = 1$, $A = 2$.)

EQ.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.25	0.242+	0.217—	0.165+
0.5	0.484+	0.433+	0.331—
0.75	0.726+	0.650—	0.496+
1.	0.968+	0.866+	0.661+
1.25	1.210+	1.083—	0.827—
1.5	1.452+	1.299+	0.992+
1.75	1.694+	1.516—	1.157+
2.	1.936+	1.732+	1.323—

19. *Thirdly*; If it be cut by $c\Sigma\sigma q$, a Plain parallel to the Rectangle CDBA, *Fig.* 2. the Curve Line will have this property: Drawing the Triangles as in the Scheme, it is, *As* SR (the Ordinate from any Point S in the Arch ΣQ) *to* $\sigma\varrho$, or ΣP (the Ordinate from Σ, where the Plain $c\Sigma q$ cuts the Quadrantal Arch): *So* is aR or AC (the whole height), *to* $a\varrho$ or $\alpha\sigma$ (the distance of the Point σ from the Plain $Aa\alpha$ parallel to the Quadrant CDQ). Because aRS, $a\varrho\sigma$, are like Triangles.

20. The Quantities I thus design in *Species*.

$$CQ = CD = R.$$
$$CP = C.$$
$$CR = c.$$
$$P\Sigma = \varrho\sigma = \sqrt{:R^2 - C^2}.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$Ra = CA = A.$$
$$RS \,.\, Ra :: \varrho\sigma\ (= P\Sigma) \,.\, \sigma\alpha = \varrho a.$$
$$\sqrt{:R^2 - c^2} \,.\, A :: \sqrt{:R^2 - C^2} \,.\, A\sqrt{\frac{R^2 - C^2}{R^2 - c^2}}.$$
$$\sigma\alpha = \varrho a = A\sqrt{\frac{R^2 - C^2}{R^2 - c^2}}.$$

Fig. 2.

21. These Curve Lines (if made by Plains at equal distances) projected on the Rectangle CDBA (to which they are parallel), will appear as in the third Projection, *Fig.* 18. which is thus made: Having drawn a Rectangle CDBA (like and equal to that in the Solid), and the Quadrant CDQ; divide CQ into any number of equal parts at the Points c, and draw the Sines or Ordinates $c\Sigma$, with the Co-sines ΣP: Then supposing from the several Points R in the Line CD, the Lines $R\sigma a$ parallel to CA, (cutting the Quadrant QD at S, and AB at a:) And therein, *As* RS *to* $P\Sigma$; *So* AC (= aR) *to* $a\sigma$: The Curve Lines $q\sigma P$ in this Plain, represent their Respectives $q\sigma S$ in the Solids. Where note, That as the Lines $S\sigma\sigma\sigma$ in the former Projection, so are the Lines $a\sigma\sigma\sigma$ in this cut into equal parts.

22. As the Solid may be continued downwards at pleasure, beyond its Quadrantal Base CDQ; so may these Curve Lines $q\sigma P$, in like manner, be so continued infinitely: And they will then be *Assymptotes*, each to other; and to the streight Line BD so continued. And if the Solid be continued upward after a decussa-

decuſſation in A B, the ſame Plains will cut off in the oppoſite Solid oppoſite Sections like to theſe.

23. The Quantities in this Projection I thus deſign in *Species*.

Fig. 18.

$$\begin{aligned}
&CD = R.\\
&CP = C.\\
&CR = c.\\
&\Sigma P = S = \surd: R^2 - C^2.\\
&SR = s = \surd: R^2 - c^2.\\
&AC = A.\\
&SR \,.\, \Sigma P :: AC\,(=aR) \,.\, a\sigma.\\
&\quad s \,.\, S :: A \,.\, \frac{S}{s} A.\\
&a\sigma = \frac{S}{s} A = A \surd \frac{R^2 - C^2}{R^2 - c^2} = \frac{\surd: R^2 - C^2}{\surd: R^2 - c^2} A.
\end{aligned}$$

24. In Numbers thus; (putting $R = 1$, $A = 2$.)

	Aq.	aσ.	aσ.	aσ.	aσ.
I.	0.5	0.516+	0.577+	0.756—	*Infin.*
II.	1.	1.033—	1.155—	1.512—	*Infin.*
III.	1.5	1.549+	1.732+	2.268—	*Infin.*
IV.	2.	2.064—	2.310—	3.024—	*Infin.*

25. *Fourthly*; If it be cut by a Plain CQD, *Fig.* 3. perpendicular to the Rectangle CDBA, and paſſing by the Center C and any Point d in the Side DB, the Curve Line will have this property: Cutting this Plain in ρσ (by any of the Triangles RSa, parallel to CQA); then is, *As* AC or aR, *to* aϱ; *So* RS, to ρσ. The length of aϱ being firſt found in this manner: *As* CD *to* CR, or Cd *to* Cϱ; *So* is dD *to* ρR: Which ſubducted from the whole height, (or added thereunto, if we ſuppoſe ρ to be taken in the continuation of dC beyond C) gives the length of aϱ.

26. The Quantities of this Section I thus deſign in *Species*.

Fig. 3, 4.

$$\begin{aligned}
&CD = R.\\
&CR = c.\\
&RS = \surd: R^2 - c^2.\\
&AC = aR = A.\\
&dD = b.\\
&CD \,.\, CR :: Cd \,.\, C\rho :: dD \,.\, \rho R.\\
&\quad R \,.\, c \qquad\qquad :: b \,.\, \frac{c}{R} b.\\
&\varrho R = \frac{c}{R} b.\\
&a\varrho = aR \mp R\varrho = A \mp \frac{c}{R} b.\\
&aR\,(=AC) \,.\, a\varrho :: RS \,.\, \varrho\sigma.\\
&A \,.\, A \mp \frac{c}{R} b :: \surd: R^2 - c^2 \,.\, \frac{AR \mp cb}{AR} \surd: R^2 - c^2.\\
&\rho\sigma = \frac{AR \mp cb}{AR} \surd: R^2 - c^2.
\end{aligned}$$

27. But if it be cut by a Plain CQb, *Fig.* 4. which paſſing by CQ, cuts any Point b (in the Side AB) before it come at d in the Side DB produced; the Curve Line bσQ will have this property: Cutting this Plain, as before, in ϱσ, by any of the Triangles RSa parallel to CQA; then is, *As* bA *to* ba; or, *As* bC *to* bϱ; or, *As* AC (or aR) *to* aρ: *So* RS *to* ρσ.

28. In

28. In *Species*, thus:

$CD = R.$
$CR = c.$
$RS = \sqrt{:R^2 - c^2}.$ *Fig.4.*
$AC = A.$
$bA = V = C.$
$ba = v = C - c.$
$bA \,.\, ba :: bC \,.\, b\varrho :: AC\,(=aR) \,.\, a\varrho :: RS \,.\, \varrho\sigma.$
$V \,.\, v :: \sqrt{:R^2 - c^2} \,.\, \frac{v}{V}\sqrt{:R^2 - c^2}.$

$\varrho\sigma = \frac{v}{V}\sqrt{:R^2 - c^2}. = \frac{C-c}{C}\sqrt{:R^2 - c^2}.$

29. Or elſe, continuing Cb till it cut DB (continued) in d, the Proportions will be as before, at § 25, 26.

30. But with this difference; That the Curve $Q\sigma bd$ will cut its Axis at b, and meet with it again at d, (the part bd being on the other ſide of the Axis, and of the Plain ABDC, in the oppoſite Solid.) And accordingly (Rϱ being in this caſe greater than aR) the Quantities aϱ, and $\rho\sigma$, will be Negative Quantities; aρ falling beyond the Vertex AB, which was ſuppoſed ſhort of it; and $\rho\sigma$ below the Plain ABDC, which was ſuppoſed above it.

31. In both theſe caſes (whether Cd cut or cut not the Vertex AB) the Lines dC continued (anſwering to a ſutable continuation of the Solid) will again meet with their Axes continued at δ (as far beyond C, as d is on this ſide it). But the Ordinates in this continuation will be greater than thoſe of dC; becauſe from CD upward the Solid grows thinner, but thicker from CD downward. And accordingly, aϱ, which between AB and CD is leſs than aR, (and above AB, a Negative Quantity;) the ſame below CD becomes greater than aR; (dC cutting DC at C:) For there it is $a\varrho = aR - R\varrho$; here it is $a\varrho = aR + R\varrho$.

32. Theſe Curves, in both caſes, (when Cd cuts or cuts not the Line AB) ſuppoſing the Side DB divided into equal parts by the Lines Cd, (if projected on one and the ſame Plain) will appear as in the fourth Projection, *Fig.* 19. Where the Axes dC being continued to δ, are then (to avoid confuſion in the Figure) removed from their proper place in the Plain ABDC, and ſet off in the ſame ſtreight Line AC continued; and the Ordinates $\varrho\sigma$ applied to them in that Poſition, in ſuch proportion to RS, as aρ is to aR or AC: And moreover, they are ſo diſtributed, ſome on the one ſide, ſome on the other ſide of AD, to prevent the confuſion which might ariſe in the Figure, if ſo many Curves ſhould all interſect one another in the ſame Point Δ, beſide another interſection afterwards.

33. The Quantities in this Projection, I thus deſign in *Species*.

$CD = R.$
$CR = c.$
$RS = \sqrt{:R^2 - c^2}.$
$AC = A.$ *Fig.19.*
$dD = b.$
$CD \,.\, CR :: Dd \,.\, R\rho.$
$R \,.\, c :: b \,.\, \frac{cb}{R}.$
$R\varrho = \frac{cb}{R}.$
$a\varrho = A \mp \frac{cb}{R}.$
$AC \,.\, a\varrho :: RS \,.\, \varrho\sigma.$
$A \,.\, A \mp \frac{cb}{R} :: \sqrt{:R^2 - c^2} \,.\, \frac{AR \mp cb}{AR}\sqrt{:R^2 - c^2}.$
$\rho\sigma = \frac{AR \mp cb}{AR}\sqrt{:R^2 - c^2}.$

34. In Numbers (putting $R=1$, $A=2$,) the Semi-axes to the six Curves described (whereof the first is the circumference of a Circle) are these:

I.	II.	III.	IV.	V.	VI.
CD	Cd.	Cd.	Cd.	Cd.	Cd.
1.	$\sqrt{1.25}$.	$\sqrt{2}$.	$\sqrt{2.25}$.	$\sqrt{5}$.	$\sqrt{7.25}$.
	1.11803	1.41421	1.80278	2.23607	2.69258

And the Ordinates, supposing the Semi-axes divided into four parts, are these:

I.	II.	III.	IV.	V.	VI.
RS.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.	0.	0.	0.	0.	0.
0.661+	0.537+	0.413+	0.289+	0.165+	0.041+
0.866+	0.758—	0.650—	0.541+	0.433+	0.325—
0.968+	0.908—	0.847—	0.787—	0.726+	0.666—
1.	1.	1.	1.	1.	1.
0.968+	1.029—	1.089—	1.150—	1.210+	1.271—
0.866+	0.974+	1.083—	1.194+	1.299+	1.407+
0.661+	0.785+	0.909+	1.033+	1.158—	1.282—
0.	0.	0.	0.	0.	0.

Or if (for a more accurate describing of the Curves) the Semi-axes be divided into 16 equal parts (and the whole Axes into 32), the Ordinates thereunto appertaining are these.

I.	II.	III.	IV.	V.	VI.
RS.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.	0.	0.	0.	0.	0.
0.3481	0.2665	0.1849	0.1033	0.0217	—0.0598
0.4841	0.3782	0.2723	0.1664	0.0605	—0.0454
0.5830	0.4645	0.3461	0.2277	0.1093	—0.0091
0.6614	0.5374	0.4134	0.2894	0.1654	+0.0413
0.7262	0.6014	0.4765	0.3517	0.2269	0.1021
0.7806	0.6586	0.5367	0.4147	0.2921	0.1702
0.8268	0.7105	0.5943	0.4780	0.3617	0.2454
0.8660	0.7578	0.6495	0.5413	0.4330	0.3248
0.8992	0.8009	0.7025	0.6042	0.5058	0.4074
0.9270	0.8401	0.7532	0.6663	0.5794	0.4925
0.9499	0.8757	0.8015	0.7273	0.6531	0.5789
0.9682	0.9077	0.8472	0.7867	0.7262	0.6657
0.9823	0.9362	0.8902	0.8441	0.7981	0.7520
0.9922	0.9612	0.9301	0.8991	0.8681	0.8371
0.9980	0.9824	0.9668	0.9513	0.9357	0.9201
1.	1.	1.	1.	1.	1.
0.9980	1.0136	1.0292	1.0448	1.0604	1.0760
0.9922	1.0232	1.0542	1.0852	1.1162	1.1472
0.9823	1.0283	1.0743	1.1204	1.1664	1.2125
0.9682	1.0288	1.0893	1.1498	1.2103	1.2708
0.9499	1.0241	1.0983	1.1726	1.2468	1.3210
0.9270	1.0139	1.1008	1.1877	1.2746	1.3616
0.8992	0.9976	1.0959	1.1943	1.2926	1.3910
0.8660	0.9743	1.0825	1.1908	1.2990	1.4073
0.8268	0.9431	1.0593	1.1756	1.2919	1.4081
0.7806	0.9026	1.0246	1.1465	1.2679	1.3899
0.7262	0.8510	0.9758	1.1006	1.2254	1.3502
0.6614	0.7854	0.9095	1.0335	1.1575	1.2815
0.5830	0.7014	0.8198	0.9382	1.0566	1.1750
0.4841	0.5900	0.6959	0.8018	0.9077	1.0136
0.3481	0.4297	0.5112	0.5928	0.6744	0.7560
0.	0.	0.	0.	0.	0.

35. *Fifthly*;

35. *Fifthly*; If it be cut by a Plain $D\sigma qc$, *(Fig. 5.)* passing through D, and perpendicular to the Rectangle ABD, cutting AC in any Point c, and AQ in q; the Curve Line $D\sigma q$ will have this property: Cutting this Plain by any of the Triangles RSa in $\varrho\sigma$, it will be, *As* AC, or aR, *to* RS; *So* $a\varrho$ *to* $a\sigma$. The length of $a\rho$ being first found thus: *As* DC *to* DR, or Dc *to* $D\rho$; *So* is Cc *to* $R\varrho$: Which subducted from aR, leaves $a\rho = aR - R\rho$; and here $DR = DC - CR$: But if Dc be supposed to be continued beyond c, and consequently R fall beyond c, then is $DR = DC + CR$.

36. The Quantities of this Section I thus design in *Species*.

$$CD = R.$$
$$CR = c.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$aR = AC = A.$$
$$Cc = b.$$
$$DC \,.\, DR :: Dc \,.\, D\rho :: Cc \,.\, R\varrho.$$
$$R \,.\, R \mp c :: \qquad b \,.\, \frac{R \mp c}{R} b.$$
$$R\varrho = \frac{R \mp c}{R} b.$$
$$a\varrho = aR - R\varrho = A - \frac{R \mp c}{R} b.$$
$$aR \,.\, a\varrho :: RS \,.\, \varrho\sigma.$$
$$A \,.\, A - \frac{R \mp c}{R} b :: \sqrt{:R^2 - c^2} \,.\, \frac{AR - bR \pm bc}{AR} \sqrt{:R^2 - c^2}.$$
$$\rho\sigma = \frac{AR - bR \pm bc}{AR} \sqrt{:R^2 - c^2}.$$

Fig. 5, 6.

37. But if this Plain (passing by D) cut any Point b in the Line AB, before it come at c in the continuation of CA, *(Fig. 6.)* the Curve Line will have this property: Cutting this Plain (as before) in $\rho\sigma$, by any of the Triangles RSa parallel to CQA, then is, *As* bB *to* ba; or, *As* bD *to* $b\rho$; or, *As* BD (or aR) *to* $a\varrho$: *So* is RS *to* $\varrho\sigma$.

38. In *Species*, thus:

$$CD = R.$$
$$CR = Aa = c.$$
$$Ab = C.$$
$$AC = aR = A.$$
$$bB = V = R - C.$$
$$ba = v = c - C.$$
$$bB \,.\, ba :: bD \,.\, b\varrho :: BD\ (= AC = aR) \,.\, a\varrho :: RS \,.\, \varrho\sigma.$$
$$V \,.\, v :: \qquad A \,.\, \frac{v}{V} :: \sqrt{:R^2 - c^2} \,.\, \frac{v\sqrt{:R^2 - c^2}}{V}.$$
$$a\rho = \frac{v}{V} A = \frac{c - C}{R - C} A.$$
$$\varrho\sigma = \frac{v}{V} \sqrt{:R^2 - c^2}. = \frac{c - C}{R - C} \sqrt{:R^2 - c^2}.$$

Fig. 6.

39. Or else, continuing Db till it cut CA (continued) in c, the Proportions will be as before, at § 35, 36.

40. But with this difference; That the Curve $D\sigma bq$ will cut its Axis at b, (the part bq being on the other side of the Axis, and of the Plain ABDC, in the opposite Solid.) And accordingly ($R\varrho$ being in this case greater than aR) the Quantities $a\varrho$, and $\rho\sigma$, will be Negative Quantities; $a\rho$ falling beyond the Vertex AB, (which was supposed short of it) and $\rho\sigma$ below the Plain ABDC, which was supposed above it.

41. In both these cases (whether Dc cut or cut not the Vertex AB) the Curve Lines $D\sigma c$ continued (answering to a sutable continuation of the Solid) will again meet with their Axes (continued) at δ (as far beyond c, as D is on this side it.)

it). And if, in the mean time, the Axis D δ cut the Vertex B A, or its continuation beyond A, the Curve will, in the ſame point, cut its Axis, and (paſſing thenceforth on the other ſide) meet with it again at δ.

42. Theſe Curves, in both caſes, (whether D c cut or cut not the Line B A, or its continuation) ſuppoſing the Line C A divided into equal parts by the Lines D c, (projected on the ſame Plain) will appear as in the fifth Projection, *Fig.* 20. Where the Lines D c being continued to δ, are (to avoid confuſion in the Figure) removed from their proper place in the Plain A B D C, and all ſet off in the ſame ſtreight Line C A continued; and the Lines $\varrho\sigma$ are applied to them as Ordinates (in this Poſition) in ſuch proportion to R S, as a ρ is to a R.

43. The Quantities in this Projection I thus deſign in *Species*.

Fig. 20.

$$C D = R.$$
$$C R = c.$$
$$R S = \sqrt{:} R^2 - c^2.$$
$$a R = A C = A.$$
$$C c = b.$$
$$D C . D R :: D c . D \rho :: C c . R \varrho.$$
$$R . R \mp c :: b . \frac{R \mp c}{R} b.$$
$$R \varrho = \frac{R \mp c}{R} b.$$
$$a \varrho = a R - R \varrho = A - \frac{R \mp c}{R} b.$$
$$a R . a \varrho :: R S . \varrho \sigma.$$
$$A . A - \frac{R \mp c}{R} b :: \sqrt{:} R^2 - c^2 . \frac{AR - bR + bc}{AR} \sqrt{:} R^2 - c^2.$$
$$\rho \sigma = \frac{AR - bR + bc}{AR} \sqrt{:} R^2 - c^2.$$

44. In Numbers (putting $R = 1$, $A = 2$,) the Semi-axes of the ſix Curves deſcribed, D c, are of the ſame length with C d, § 34. And the Ordinates, ſuppoſing the Semi-axes divided into four equal parts, are theſe:

I.	II.	III.	IV.	V.	VI.
R S.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.	0.	0.	0.	0.	0.
0.661	0.620	0.579	0.537	0.496	0.455
0.865	0.758	0.650	0.541	0.433	0.325
0.968	0.787	0.605	0.424	0.242	0.061
1.	0.75	0.5	0.25	0.	—0.25
0.968	0.666	0.363	0.6 0	—0.242	—0.545
0.866	0.541	0.217	—0.108	—0.433	—0.758
0.661	0.372	0.083	—0.207	—0.496	—0.785
0.	0.	0.	0.	0.	0.

Or (for a more accurate deſcribing the Curves) dividing the Semi-axe into 16 parts (and the whole Axis into 32), the Ordinates will be theſe.

I.

I. RS.	II. ρσ.	III. ρσ.	IV. ρσ.	V. ρσ.	VI. ρσ.
0.	0.	0.	0.	0.	0.
0.3481	0.3426	0.3372	0.3318	0.3263	0.3209
0.4841	0.4690	0.4539	0.4387	0.4236	0.4085
0.5830	0.5556	0.5283	0.5010	0.4736	0.4461
0.6614	0.6201	0.5787	0.5374	0.4960	0.4547
0.7262	0.6696	0.6130	0.5564	0.4998	0.4452
0.7806	0.7075	0.6343	0.5611	0.4880	0.4148
0.8268	0.7364	0.6459	0.5555	0.4651	0.3747
0.8660	0.7578	0.6495	0.5413	0.4330	0.3248
0.8992	0.7728	0.6463	0.5199	0.3934	0.2670
0.9270	0.7822	0.6373	0.4925	0.3476	0.2028
0.9499	0.7866	0.6234	0.4601	0.2969	0.1336
0.9682	0.7867	0.6052	0.4236	0.2421	0.0605
0.9823	0.7827	0.5832	0.3837	0.1842	—0.0153
0.9922	0.7750	0.5581	0.3411	0.1240	—0.0930
0.9980	0.7641	0.5302	0.2963	0.0624	—0.1715
1.	0.75	0.5	0.25	0.	—0.25
0.9980	0.7329	0.4678	0.2027	—0.0624	—0.3275
0.9922	0.7131	0.4341	0.1550	—0.1240	—0.4031
0.9823	0.6906	0.3990	0.1074	—0.1842	—0.4758
0.9682	0.6657	0.3631	0.0605	—0.2421	—0.5446
0.9499	0.6382	0.3265	0.0148	—0.2969	—0.6085
0.9270	0.6084	0.2897	—0.0290	—0.3476	—0.6663
0.8992	0.5761	0.2529	—0.0713	—0.3934	—0.7166
0.8660	0.5413	0.2165	—0.1083	—0.4330	—0.7578
0.8268	0.5038	0.1809	—0.1421	—0.4651	—0.7880
0.7806	0.4635	0.1463	—0.1708	—0.4880	—0.8051
0.7262	0.4197	0.1132	—0.1933	—0.4998	—0.8043
0.6614	0.3721	0.0827	—0.2007	—0.4960	—0.7854
0.5830	0.3188	0.0546	—0.2095	—0.4736	—0.7378
0.4841	0.2572	0.0303	—0.1967	—0.4236	—0.6505
0.3481	0.1795	0.0109	—0.1577	—0.3263	—0.4949
0.	0.	0.	—0.	—0.	—0.

45. *Sixthly*; If it be cut by a Plain B q c, passing through B (*Fig.* 7.) perpendicular to the Rectangle A B D C, and cutting the Side A C in c, (and any of the Triangles R S a in ρσ) the Curve Line will have this property: *As* A C, or a R, *to* a ρ; *So* is R S, *to* ρσ. The length of a ρ being first found thus: *As* B A *to* B a, or B c *to* B ρ; *So* is A c *to* a ρ.

46. In *Species*, thus:

Fig. 7.

$$AB = CD = R.$$
$$Aa = CR = c.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$aR = AC = A.$$
$$Ac = a.$$

$$BA \,.\, Ba :: Bc \,.\, B\rho :: Ac \,.\, a\rho.$$
$$R \,.\, R \mp c :: \qquad a \,.\, \frac{R \mp c}{R} a.$$
$$a\rho = \frac{R \mp c}{R} a.$$
$$aR\ (=AC) \,.\, a\rho :: RS \,.\, \rho\sigma.$$
$$A \,.\, \frac{R \mp c}{R} a :: \sqrt{:R^2 - c^2} \,.\, \frac{R \mp c}{AR} a\sqrt{:R^2 - c^2}.$$
$$\rho\sigma = \frac{R \mp c}{AR} a\sqrt{:R^2 - c^2}.$$

47. But

47. But if this Plain (paſſing by B) cut the Line DC in any Point P, before it come at c in the continuation of AC, the Curve Line will have this property: Cutting the ſame Plain (as before) by any of the Triangles RSa parallel to CQA, it will be, *As* DP *to* DR; or, *As* BP *to* Bρ; or, aR (=AC) *to* aϱ; *So* RS *to* ρσ.

48. In *Species*, thus:

Fig.7.

$CD = R.$
$CP = C.$
$CR = c.$
$RS = \sqrt{:R^2 - c^2}.$
$DP = R - C = V.$
$DR = R - c = v.$
$AC = A.$

$DP . DR :: BP . B\varrho :: aR\ (=BD=AC) . a\varrho :: RS . \varrho\sigma.$

$R-C . R-c :: V . v :: A . \frac{R-c}{R-C}A = \frac{v}{V}A :: \sqrt{:R^2-c^2} . \frac{R-c}{R-C}\sqrt{:R^2-c^2}.$

$a\varrho = \frac{v}{V}A = \frac{R-c}{R-C}.$

$\varrho\sigma = \frac{v}{V}\sqrt{:R^2-C^2}. = \frac{R-c}{R-C}\sqrt{:R^2-c^2}.$

49. Or elſe, continuing DP till it cut AC (continued) in c, the Proportions will be as before at §45, 46.

50. But in this Section, the Curve doth not cut its Axis at P (as in the two Sections laſt mentioned at b), but continues on the ſame ſide of it, till it meet again (if it be continued) at β. And (in caſe of ſuch continuation) inſtead of $DR = R - c$, it will be $DR = R + c$, where the Point R is beyond the Line AC: And, in like manner, if after Bc have cut AC in c, it cut the continuation of AB in P, then, inſtead of $DP = R - C$, it will be $DP = R + C$.

51. In both caſes (whether Bc cut AC above or below the Point C) the Curve Lines Bσc continued (anſwering to a ſutable continuation of the Solid) will again meet with their Axes (continued) at β, as far beyond c as B is on this ſide it; but continues all on the ſame ſide of its Axis, without cutting it in the way, as in the two laſt mentioned Sections.

52. In both caſes (whether Bc cut or cut not DC) the Curve Lines Bσcβ transferred to one and the ſame Plain, (ſuppoſing the Line AC divided into equal parts at the Points c) will appear as in the ſixth Projection, *Fig.* 21. where the Lines Bc are continued to β, and then (to avoid confuſion in the Figure) removed from their proper place in the Plain ABDC, and all ſet off in the ſame ſtreight Line AC (continued), and the Lines ϱσ applied to them as Ordinates (in this Poſition) in ſuch proportion to RS, as aρ to aR.

53. The Quantities in this Projection, I thus deſign in *Species*.

Fig.21.

$AB = CD = R.$
$Aa = CR = c.$
$RS = \sqrt{:R^2 - c^2}.$
$DR = R \mp c = v.$
$aR = AC = A.$
$Ac = a.$

$BA . Ba :: Bc . B\varrho :: Ac . a\varrho.$

$R . R \mp c = v :: a . \frac{R \mp c}{R}a = \frac{v}{R}a.$

$a\varrho = \frac{R \mp c}{R}a = \frac{v}{R}a.$

$aR\ (=AC) . a\varrho :: RS . \varrho\sigma.$

$A . \frac{R \mp c}{R}a :: \sqrt{:R^2-c^2} . \frac{R \mp c}{R}a\sqrt{:R^2-c^2}.$

$\varrho\sigma = \frac{R \mp c}{R}a\sqrt{:R^2-c^2} = \frac{v\,a}{RA}\sqrt{:R^2-c^2}.$

54. In

54. In Numbers (putting $R = 1$, $A = 2$,) the Semi-axes of the five Curves described, Bc, are of the same length with Cd, and Dc, § 34, 44. save that the first of those (which is the circumference of a Circle) is here omitted; (instead of which, in this case, we should have a streight Line, coincident with its Axe BA.)

I.	II.	III.	IV.	V.
Bc.	Bc.	Bc.	Bc.	Bc.
1.11803	1.41421	1.80278	2.23607	2.69258

And the Ordinates, supposing the Semi-axis divided into four parts, are these:

I.	II.	III.	IV.	V.
$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.	0.	0.	0.	0.
0.041	0.083	0.124	0.165	0.207
0.108	0.217	0.325	0.433	0.541
0.182	0.363	0.545	0.726	0.908
0.25	0.5	0.75	1.	1.25
0.303	0.605	0.908	1.210	1.513
0.325	0.650	0.974	1.299	1.624
0.289	0.579	0.868	1.157	1.447
0.	0.	0.	0.	0.

Or (for a more accurate describing the Curve) dividing the Semi-axe into 16 parts (and the whole Axe into 32), the Ordinates will be these:

I.	II.	III.	IV.	V.
$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.	$\varrho\sigma$.
0.	0.	0.	0.	0.
0.0054	0.0109	0.0163	0.0218	0.0272
0.0151	0.0303	0.0454	0.0605	0.0756
0.0273	0.0547	0.0820	0.1093	0.1366
0.0414	0.0827	0.1241	0.1654	0.2068
0.0566	0.1132	0.1698	0.2264	0.2830
0.0732	0.1463	0.2195	0.2927	0.3658
0.0904	0.1809	0.2713	0.3617	0.4521
0.1083	0.2165	0.3248	0.4330	0.5413
0.1265	0.2529	0.3794	0.5058	0.6323
0.1448	0.2897	0.4345	0.5794	0.7242
0.1633	0.3265	0.4898	0.6531	0.8163
0.1815	0.3631	0.5446	0.7262	0.9077
0.1995	0.3990	0.5986	0.7981	0.9976
0.2170	0.4341	0.6511	0.8681	1.0852
0.2339	0.4678	0.7018	0.9357	1.1696
0.25	0.5	0.75	1.	1.25
0.2651	0.5302	0.7953	1.0604	1.3255
0.2790	0.5581	0.8371	1.1162	1.3952
0.2916	0.5832	0.8748	1.1664	1.4581
0.3026	0.6052	0.9077	1.2103	1.5129
0.3117	0.6234	0.9351	1.2468	1.5585
0.3187	0.6373	0.9560	1.2747	1.5933
0.3232	0.6463	0.9695	1.2926	1.6158
0.3248	0.6495	0.9743	1.2990	1.6238
0.3230	0.6459	0.9689	1.2919	1.6148
0.3171	0.6343	0.9514	1.2686	1.5857
0.3065	0.6130	0.9195	1.2260	1.5325
0.2894	0.5787	0.8681	1.1575	1.4468
0.2641	0.5283	0.7925	1.0566	1.3208
0.2269	0.4539	0.6808	0.9077	1.1347
0.1686	0.3372	0.5058	0.6744	0.8430
0.	0.	0.	0.	0.

55. *Seventhly;*

55. *Seventhly*; If it be cut by a Plain A σ d, passing through A (*Fig.* 8.) perpendicular to the Rectangle A B D C, and cutting the Side B D in d, (and any of the Triangles R S a in ρ σ) the Curve Line d σ A will have this property: *As* A C, or a R, *to* a ρ; *So* is R S, *to* ρ σ. And a ρ is thus found: *As* A B *to* A a, or A D *to* A ρ; *So* is B d *to* a ρ.

56. In *Species*, thus:

Fig. 8.

$$AB = CD = R.$$
$$Aa = CR = c.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$aR = AC = A.$$
$$Bd = a.$$
$$AB \,.\, Aa :: CD \,.\, CR :: Bd \,.\, a\varrho.$$
$$R \,.\, c :: a \,.\, \frac{ca}{R}.$$
$$a\varrho = \frac{ca}{R}.$$
$$aR\,(=AC) \,.\, a\varrho :: RS \,.\, \varrho\sigma.$$
$$A \,.\, \frac{ca}{R} :: \sqrt{:R^2 - c^2} \,.\, \frac{ca}{RA}\sqrt{:R^2 - c^2}.$$
$$\varrho\sigma = \frac{ca}{RA}\sqrt{:R^2 - c^2}.$$

57. But if this Plain (passing by A) cut the Line C D in any Point P before it come at d in the continuation of B D, (*Fig.* 9.) the Curve Line will have this property: Cutting the same Plain (as before) by any of the Triangles R S a parallel to C Q A, it will be, *As* C P *to* C R; or, *As* A P *to* A ρ; or, a R (= A C) *to* a ϱ; *So* R S *to* ρ σ.

58. In *Species*, thus:

Fig. 9.

$$CD = R.$$
$$CP = C.$$
$$CR = c.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$aR = AC = A.$$
$$CP \,.\, CR :: AP \,.\, AR :: aR\,(=AC) \,.\, a\varrho :: RS \,.\, \varrho\sigma.$$
$$C \,.\, c :: A \,.\, \frac{c}{C}A :: \sqrt{:R^2 - c^2} \,.\, \frac{c}{C}\sqrt{:R^2 - c^2}.$$
$$a\varrho = \frac{c}{C}A.$$
$$\varrho\sigma = \frac{c}{C}\sqrt{:R^2 - c^2}.$$

59. Or else, continuing A P till it cut B D (continued) in d, the Proportions will be as before at § 55, 56.

60. In this Section, the Curve Line d σ A cuts not its Axis at P (as in the fourth and fifth Section at b), but continues on the same side of it (above the Plain A B D C) till it meet with it at A; but (supposing the Solid to be farther continued in the opposite Position) cuts it at A, and thenceforth continues on the other side of it (below the Plain A B D C continued) till it meet again at $\sigma^{\wedge}$. And (in case of such continuation) instead of $CR = +c$, we shall have $CR = -c$; (because now R falls on the contrary side of C, in the continuation of D C:) And consequently the Ordinates beyond A (being on the contrary side) to be interpreted Negatively (with the sign —) as those on this side, Affirmatively, with the sign +.

61. In both cases (whether A d cut or cut not the Line C D as at P; that is, whether the Point d fall above or below D;) the Curve Lines d σ A continued (answering to a suitable continuation of the Solid) cutting their Axis at A, will again meet with it (continued) at $\sigma^{\wedge}$, as far beyond A as d is on this side of it: And the Ordinates beyond A will be just the same as on this side, but with contrary signs + —.

62. And

62. And the Curve Lines d σ A δ transferred to one and the ſame Plain, (ſuppoſing the Line B D divided into equal parts at the Points d) will appear as in the ſeventh Projection, *Fig.* 22. Where the Lines d A are continued to δ, and then (to avoid confuſion in the Figure) removed from their proper place in the Plain A B D C, and all ſet off in the ſtreight Line A C (continued), and the Lines ϱ σ applied to them as Ordinates (in this Poſition) in ſuch proportion to R S, as a ϱ is to a R.

63. The Quantities in this Projection I thus deſign in *Species*.

$$AB = CD = R.$$
$$Aa = CR = \pm c.$$
$$RS = \sqrt{:R^2 - c^2}.$$
$$aR = AC = A.$$
$$Bd = a.$$

Fig. 22.

$$AB \,.\, Aa :: CD \,.\, CR :: Bd \,.\, a\varrho.$$
$$R \,.\, \pm c :: a \,.\, \pm\frac{ca}{R}.$$
$$a\varrho = \frac{\pm ca}{R}.$$
$$aR\ (=AC) \,.\, a\varrho :: RS \,.\, \varrho\sigma.$$
$$A \,.\, \frac{\pm ca}{R} :: \sqrt{:R^2 - c^2} \,.\, \frac{\pm ca}{RA}\sqrt{:R^2 - c^2}.$$
$$\varrho\sigma = \frac{\pm ca}{RA}\sqrt{:R^2 - c^2}.$$

64. In Numbers (putting $R = 1$, $A = 2$,) the Semi-axes of the five Curves deſcribed, A d, are of the ſame length with B C, § 54.

I.	II.	III.	IV.	V.
A d.	A d.	A d.	A d.	A d.
1.11803	1.41421	1.80278	1.23607	2.69258

And the Ordinates, ſuppoſing the Semi-axis divided into four equal parts, are theſe:

I.	II.	III.	IV.	V.
ϱ σ.	ϱ σ.	ϱ σ.	ϱ σ.	ϱ σ.
0.	0.	0.	0.	0.
0.124	0.248	0.372	0.496	0.620
0.108	0.217	0.325	0.433	0.523
0.061	0.121	0.182	0.242	0.303
± 0.	± 0.	± 0.	± 0.	± 0.303
—0.061	—0.121	—0.182	—0.242	—0.523
—0.108	—0.217	—0.325	—0.433	—0.620
—0.124	—0.248	—0.372	—0.496	
—0.	—0.	—0.	—0.	

Or (for a more accurate deſcribing the Curve) dividing the Semi-axis into 16 parts (and the whole Axis into 32), the Ordinates will be theſe:

I.	II.	III.	IV.	V.
ϱσ.	ϱσ.	ϱσ.	ϱσ.	ϱσ.
0.	0.	0.	0.	0.
0.0816	0.1632	0.2447	0.3263	0.4079
0.1059	0.2118	0.3177	0.4236	0.5295
0.1184	0.2368	0.3552	0.4737	0.5921
0.1240	0.2480	0.3720	0.4960	0.6200
0.1250	0.2499	0.3749	0.4998	0.6248
0.1220	0.2440	0.3660	0.4880	0.6099
0.1163	0.2325	0.3488	0.4651	0.5814
0.1083	0.2165	0.3248	0.4330	0.5413
0.0984	0.1967	0.2951	0.3934	0.4918
0.0869	0.1738	0.2607	0.3476	0.4345
0.0742	0.1484	0.2226	0.2969	0.3711
0.0605	0.1210	0.1815	0.2421	0.3026
0.0460	0.0921	0.1383	0.1842	0.2302
0.0310	0.0620	0.0930	0.1240	0.1550
0.0156	0.0312	0.0468	0.0624	0.0780
± 0.	± 0.	± 0.	± 0.	± 0.
—0.0156	—0.0312	—0.0468	—0.0624	—0.0780
—0.0310	—0.0620	—0.0930	—0.1240	—0.1550
—0.0460	—0.0921	—0.1383	—0.1842	—0.2302
—0.0605	—0.1210	—0.1815	—0.2421	—0.3026
—0.0742	—0.1484	—0.2226	—0.2969	—0.3711
—0.0869	—0.1738	—0.2607	—0.3476	—0.4345
—0.0984	—0.1967	—0.2951	—0.3934	—0.4918
—0.1083	—0.2165	—0.3248	—0.4330	—0.5413
—0.1163	—0.2325	—0.3488	—0.4651	—0.5814
—0.1220	—0.2440	—0.3660	—0.4880	—0.6099
—0.1250	—0.2499	—0.3749	—0.4998	—0.6248
—0.1240	—0.2480	—0.3720	—0.4960	—0.6200
—0.1184	—0.2368	—0.3552	—0.4737	—0.5921
—0.1059	—0.2118	—0.3177	—0.4236	—0.5295
—0.0816	—0.1632	—0.2447	—0.3263	—0.4079
—0.	—0.	—0.	—0.	—0.

65. There are many other Sections which may be made of the ſame Solid; but theſe being all that were propoſed to be conſidered, I ſhall ſtay here.

66. But theſe four laſt mentioned (and divers others, though ſomewhat different from them) do all fall under one General, as ſo many Particulars of it: For the better conſideration of which, I ſhall compleat the Body (or at leaſt the half of it) which is here but Quadrantal; and imagin it farther to be continued downward (below its Circular Baſe) ſo far as ſhall be neceſſary; and continued upward (after an interſection in the Line A B) in like manner, as oppoſite Cones are wont to be conſidered.

67. Suppoſing then (*Fig.* 10.) on the Center C, and Diameter Δ D, a Circle deſcribed Δ S Q D; and C Q perpendicular to the Diameter Δ C D, dividing the Semicircle Δ Q D into two Quadrants; and (at Right Angles to the Plain of the Circle) a Rectangle Δ D B β, divided into two equal parts by the ſtreight Line C A; (and therefore, joining Q A, the Triangle C Q A will be at Right Angles to both the Plains:) And from every Point S in the Perimeter of the Circle, to the reſpective Points a in the Line B β (in Plains parallel to A C Q) the ſtreight Lines S a to be drawn, compleating on either ſide of the Rectangle a Curve Superficies Δ S D B β: Theſe with the Circle contain a Solid, which I call a *Cono-Cuneus*, made up of four ſuch Quadrantal Solids as are above deſcribed at § 1. which Solid (and its Oppoſite, made by a decuſſation in the Line B β) we ſuppoſe to be continued as far as is neceſſary.

68. If this Solid be cut by a Plain at Right-Angles, to the Rectangle Δ D B β, the Section of that Plain, with this Rectangle, will be either parallel to B D, (and then

then the Section will be a Right-angled Triangle, as in our firſt Caſe, § 6.) or parallel to D Δ; (and then the Section will be an Ellipſe, as in our ſecond Caſe, § 13.) or at leſt will obliquely cut the two oppoſite Sides ß Δ, B D, (produced, if need be) in ♂, d; which Line ♂ d I call, the Diameter of the Curve Line, made by the Section of the Solid.

69. And, under this laſt caſe, fall the four laſt of theſe before-mentioned, (the fourth, fifth, ſixth, and ſeventh) as appears by the Scheme: Where C d, C B, C b, anſwer the fourth Caſe; Δ b, Δ c, Δ A, anſwer the fifth Caſe; B c, B C, B r, anſwer the ſixth caſe; and A ♂, A Δ, A r, anſwer the ſeventh Caſe. But the Curves anſwering theſe Diameters I have omitted, to avoid confuſion in the Figure.

70. Now a Point being aſſigned (*Fig.* 11.) in any of the Diameters ♂ d, the Ordinate, or perpendicular height of the Curve over that Point, is thus found Geometrically: By the Point aſſigned ϱ, ſuppoſe a ρ R drawn parallel to B D, cutting ß B in a, and Δ D in R; on which, ſuppoſe a Perpendicular Plain erected, cutting the Semicircle Δ S D in R S, and the Solid in a R S a Right-lined Triangle; wherein ρ σ being drawn parallel to R S, will be Perpendicular to the Plain of the Rectangle, as is the Line R S: And therefore σ is that Point of the Curve-Superficies which is over the Point ρ, (through which therefore the Curve-Line paſſeth, whoſe Axis paſſeth through ϱ:) And therefore ϱ σ is the Ordinate to that Point of the Axis or Diameter ♂ d.

71. Therefore the Point ρ being given, a and R be known alſo; and conſequently, R S the Ordinate or Right-Sine belonging to the Point R, in the Semicircle Δ S D: Then, becauſe a R S, a ϱ σ, are like Triangles, *As* a R *is* to a ϱ, *So* is R S *to* ρ σ.

72. Having thus found as many of the Points σ as ſhall be thought neceſſary, a Curve-Line regularly drawn by them is that Curve to which ♂ d is the Axis.

73. The Arithmetical Calculation may be thus performed: Suppoſing Δ D to be divided into any number of equal parts at the Points R, the Lines R a (produced if need be) will divide ♂ d into the ſame number of equal parts at the Points ρ: And if, from d, be drawn d e parallel to Δ ♂, cutting Δ ß in e, and ρ ε from the Points ρ, the Parallels ϱ ε will cut ♂ ε into the ſame number of equal parts.

74. Suppoſing then (for the more convenient calculation) Δ D = D, Δ R = d, ♂ e = Ω, ♂ ε = ω, (and therefore ♂ d = $\sqrt{:D^2+\Omega^2}$, and ♂ ρ = $\sqrt{:d^2+\omega^2}$.) ß Δ = A (the Altitude of ß B above the Baſe), and ♂ ß = α (the Altitude of ß B the Vertex of the Solid, above ♂ the lower Vertex of the Curve-Line); then is e ß = α − Ω, ε ß = α − ω, the Altitudes of ß B above e, ε, or d, ϱ; (and therefore, when Ω, ω, happen to be greater than α, the Quantities α − Ω, α − ω, are Negatives; and conſequently, e, ε, above the Vertex ß B, in the oppoſite Solid) and R D = $D-d$; and therefore R S = $\sqrt{:D^2-d^2}$. (a mean Proportional between Δ R, R D; that is, between d, $D-d$.)

75. Then, becauſe a R S, a ρ σ, are like Triangles, (whether ϱ be higher or lower than ß B;) *As* a R, or ß Δ, *is* to a ϱ, or ß ε; *So* is R S *to* ϱ σ: That is, *As* A *to* α − ω; *So* $\sqrt{:dD-d^2}$, *to* $\frac{\alpha-\omega}{A}\sqrt{:dD-d^2}$: Which is therefore the length of ϱ σ.

76. If therefore, in a Plain, a Line equal to ♂ d, be divided at ϱ into ſo many Parts as is ſuppoſed; and on every of the Points ϱ, wherein it is ſo divided, be erected Perpendiculars equal to the Lines ϱ σ thus found reſpectively (obſerving ſtill, that the Negative Quantities are to be applied on the contrary part of the Line thus drawn); the Curve-Line drawn by the Points σ in the Plain, agrees with that in the Solid made by the Section thereof: And may be therefore deſcribed without an actual cutting of the Solid, and may be fitted to any proportion of the Height to the Baſe of the Solid; and in whatſoever poſition the Diameter ♂ d be ſuppoſed to cut the Rectangle Δ D B ß in that Solid.

77. This Calculation fitted to the Circular Baſe Δ S D, may with the ſame eaſe be applied to a *Parabola*, *Hyperbola*, or other Curve-Line whatever, whoſe Axis is Δ D, and Vertex Δ; if inſtead of $\sqrt{:dD-d^2}$, (which is here the Ordinate in the Circle) we put the Ordinate of that other Curve-Line:

As $\frac{\alpha - \omega}{A}\sqrt{:dL}$, if a *Parabola*; $\frac{\alpha - \omega}{A}\sqrt{:dL + \frac{L}{D}d^2}$, if an *Hyperbola*; (ſuppoſing D the *Tranſverſe Diameter*, and L the *Latus Rectum*); and the like in other Curves.

78. And for this reaſon, I choſe to deſign the Point R by its diſtance from Δ, rather than from C, forward and backward; and ϱ by its diſtance from δ, rather than from c; which otherwiſe might as conveniently be done.

79. But if we ſhould ſo deſign it, and put $c = RC$ (and conſequently $RS = \sqrt{:R^2 - c^2}$); and $\alpha = Ac$, the Altitude at c; and ω the difference of Altitudes at ϱ from that at c; then is $a\varrho = \alpha - \omega$ for the Points ϱ in cd above the Point c, but $a\rho = \alpha + \omega$ for thoſe below it in $c\delta$: And accordingly $\varrho\sigma = \frac{\alpha \mp \omega}{A}\sqrt{:R^2 - c^2}$.

80. But if we put $\alpha = Bd$, (the Altitude of B above d, the higher Vertex of the Curve, which therefore will be a Negative Quantity when B falls below d) and ω, the difference of Altitude from that at d, the proceſs will be the ſame as before, ſave that then, inſtead of $\alpha - \omega$, we muſt put $\alpha + \omega = \alpha\varrho$ (and conſequently $\varrho\sigma = \frac{\alpha + \omega}{A}\sqrt{:dD - d^2}$): And the like if d were the Vertex of a *Parabola*, or *Hyperbola*, or other Curve, whoſe Axis $d\delta$ ſlopeth downward.

81. If it be deſired rather to find Inſtrumentally, than by Calculation, the ſeveral Ordinates $\varrho\sigma$, to any Diameter ΔD, in any ſuch Solid, and in any Poſition aſſigned; it may be very eaſily performed in this manner.

82. Firſt; Let any ſtreight Line, at pleaſure, LM (*Fig.* 12.) be divided at the Points ϱ, into any number of unequal parts, as a Line of Ordinates at equal diſtances in the Quadrant of a Circle; (in like manner, as the Line CQ is divided at the Points S in the firſt Projection, *Fig.* 16.) and on the other ſide of M, let λM be ſo divided alſo into the ſame number of Parts; and on the ſeveral Points ϱ, M, erect Perpendiculars, continued both ways as far as ſhall be needful. Which general Conſtruction is applicable to any caſe at pleaſure; and being once drawn, may ſucceſſively be applied to many.

83. Then (ſuppoſing, in the Solid propoſed, *Fig.* 11. δE, parallel to ΔD, cutting AC in E; and Eq, parallel to CQ, cutting AQ in q;) ſet off, in the Perpendicular at M, *Fig.* 12. a Line ME equal to that Eq in the Solid, and draw the ſtreight Lines $E\lambda$, EL; which Lines will cut off, in the other Parallels, the Lines ρS, equal to the Ordinates of that Ellipſe in the Solid, (*Fig.* 11.) whoſe Axis is δE; (ſuch as are $R\sigma$ in the ſecond Projection, *Fig.* 17.)

84. In like manner, (ſuppoſing, in the Solid, *Fig.* 11. de parallel to $D\Delta$, cutting CA, or the continuation thereof, in ν; and νq, parallel to CQ, cutting QA, or the continuation thereof, in q;) ſet off in the Perpendicular at M, (*Fig.* 12, 13.) a Line $M\nu$ equal to νq; (on the ſame ſide of λML with ME, or on the contrary, according as d and δ are on the ſame, or oppoſite Sides $B\beta$;) and draw the ſtreight Lines $\nu\lambda$, νL; which Lines will cut off, in the other Parallels, the Lines ϱs, equal to the Ordinates of that Ellipſe in the Solid, whoſe Axis is νd.

85. Then (dividing $E\nu$ into as many equal parts at the Points ω, as are the unequal parts in λL) from every of the Points ω, draw ſtreight Lines to λ or L reſpectively; which Lines will cut off, in the reſpective Parallels, ϱS, (the firſt in the firſt, the ſecond in the ſecond, *&c.* numbering the Points ω from E, and the Parallels from λ;) the Lines $\varrho\sigma$, equal to the deſired Ordinates of the Curve propoſed.

86. Laſtly; Drawing a ſtreight Line δd (*Fig.* 14, 15.) equal to that in the Solid (the Diameter of the Curve propoſed); and dividing it in the Points ϱ into as many equal parts, as are the unequal parts in λL; and to each Point of Diviſion, applying at Right Angles the Lines $\rho\sigma$, equal to thoſe upon the Line λL (on the ſame or contrary ſides of δd, as thoſe are of λL); and, by the Points σ, drawing the Curve-Line which they direct: This Curve-Line is the ſame with that which is made by the Section of the Solid propoſed, by a Plain on the Line δd at Right-Angles to the Rectangle $\Delta DB\beta$.

87. The

87. The ſame may be performed by one of the Triangles λME, *Fig.* 13. reckoning the Parallels therein twice over, (the Ordinates in each Quadrant being the ſame) and dividing E*n* into as many parts as before.

88. If the Baſe ΔSD be an Ellipſe, this proceſs will be the ſame as in a Circle; but if it be a *Parabola*, *Hyperbola*, or other the like Curve, the Line λL, which is now divided as a Line of Ordinates at equal diſtances in a Circle, muſt then be divided as ſuch a Line of Ordinates of that *Parabola*, *Hyperbola*, or other Curve, whoſe Axis is ΔD: And then the reſt of the Operation purſued with very little alteration.

89. In the whole Progreſs, I have ſtill ſuppoſed the Parallelogram ABDC to be Rectangular, and the Quadrant CDQ at Right-Angles with that Plain, and the Triangles ACQ, aRS, at Right-Angles to both of them (and conſequently, the Body to be Erect, not Scalene); and the Plain cutting this Body, to be alſo at Right-Angles with that Parallelogram. But in caſe any of what we ſuppoſe to be Rectangular, ſhould be Oblique, the Sections will be ſomewhat different from theſe deſcribed, in like manner, as the Sections of Scalene Cones, or the Oblique Sections of Erect Cones, differ from the Right Sections of Right Cones. But of theſe caſes, I intend not here to diſcourſe farther; contenting my ſelf with the Perpendicular Sections of theſe Erect Solids.

FINIS.

CONO-CUNEUS.

Fig. X.

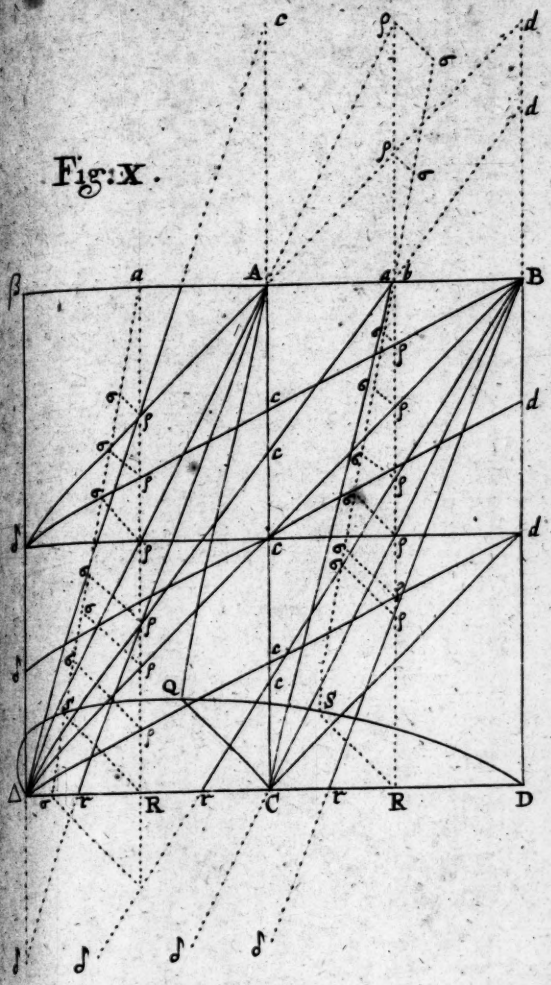


Fig. XI.

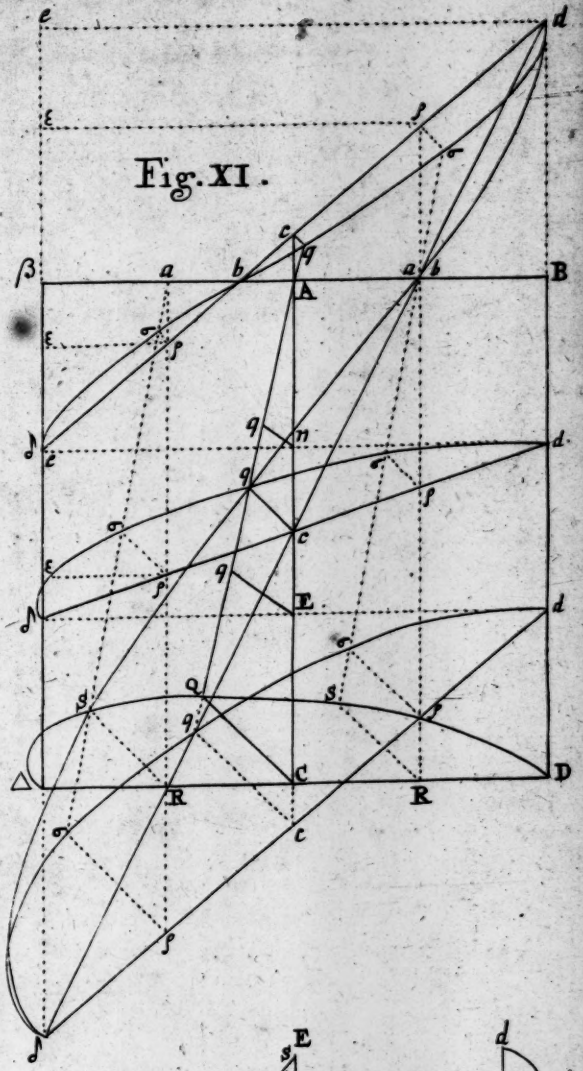


Fig. XII.

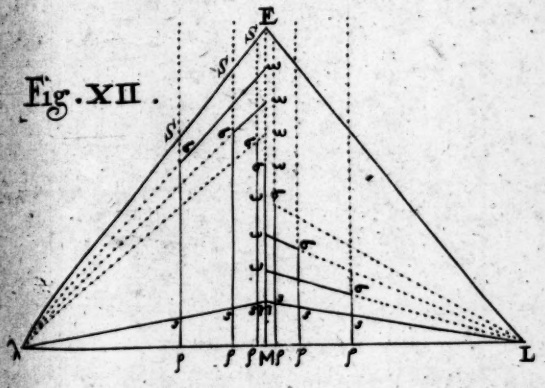


Fig. XIII.

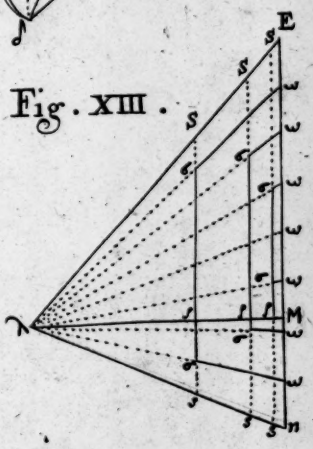


Fig. XIV.

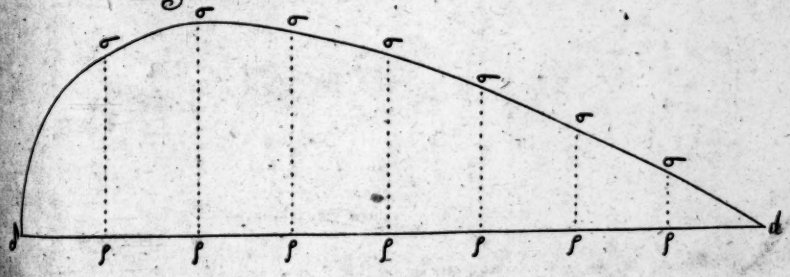


Fig. XV.

CONO-CUNEUS.

Fig. XVI.
Project. 1a.

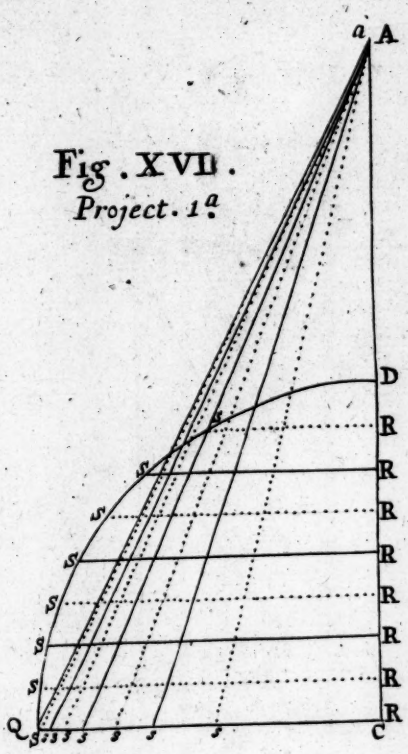


Fig. XVII.
Project. 2.

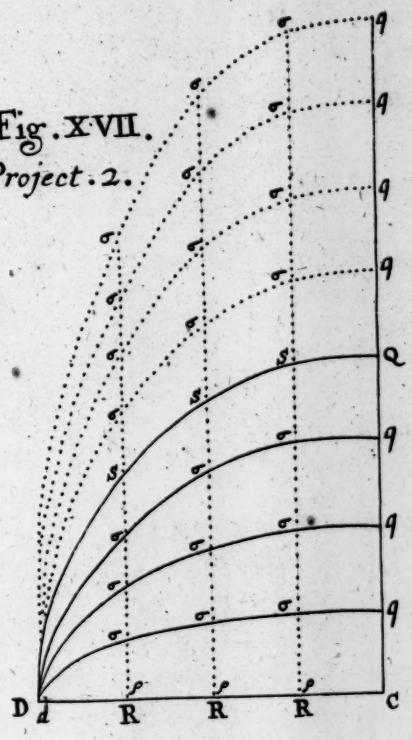
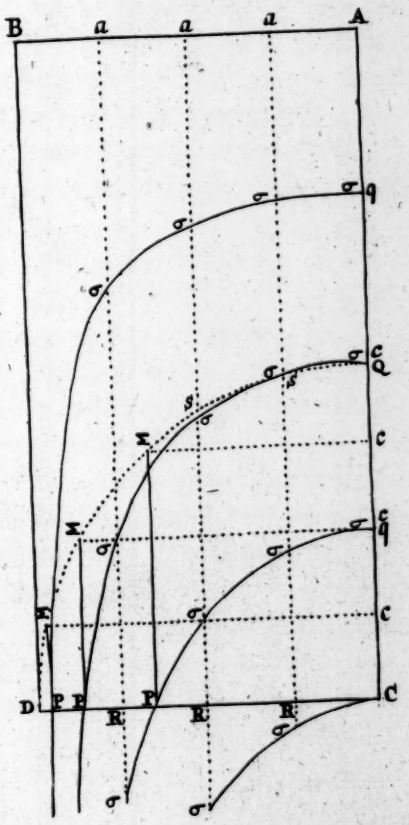
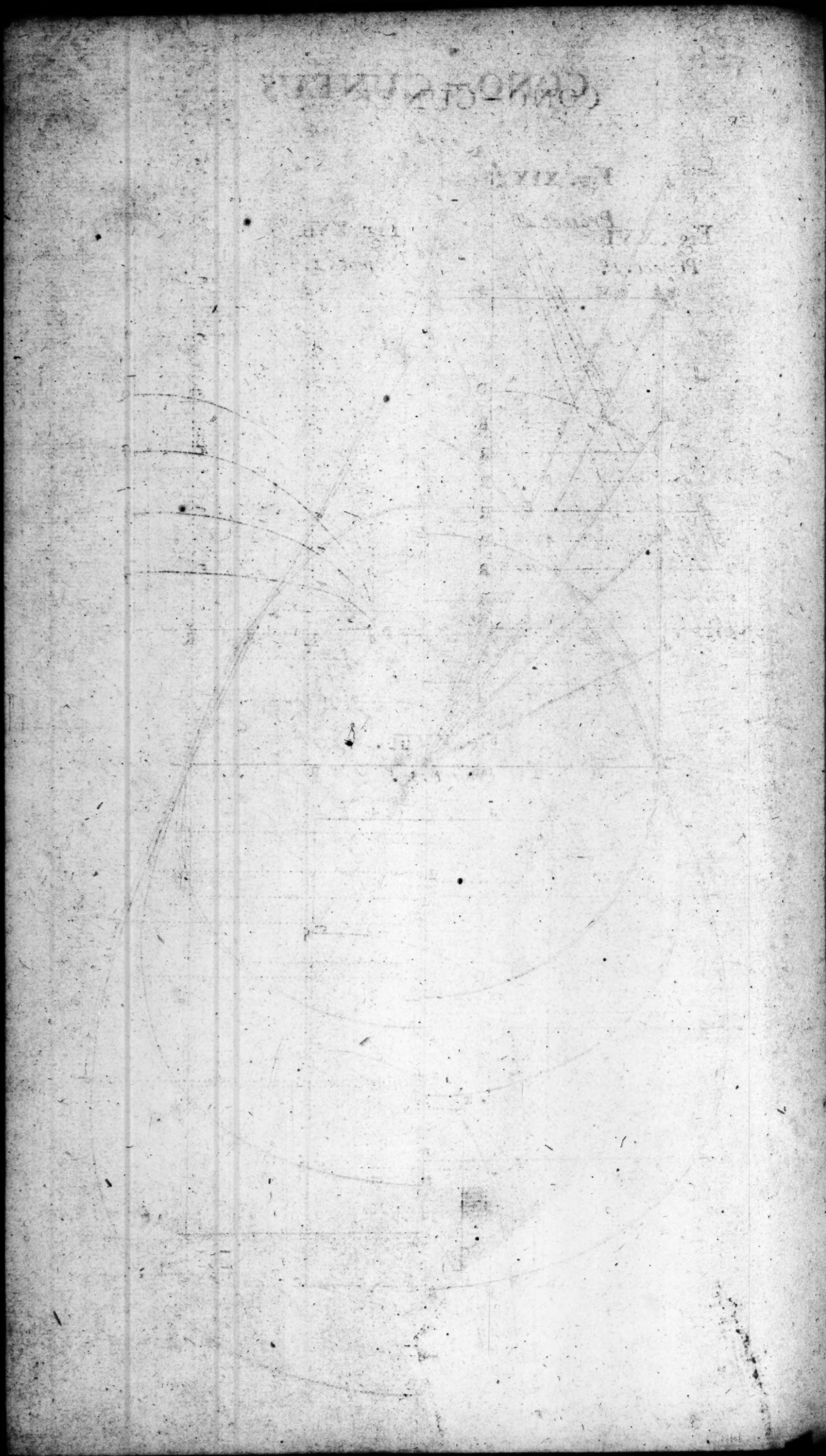


Fig. XVIII.
Project. 3.





CONO-CUNEUS.

Fig. XIX.

Project.4.a

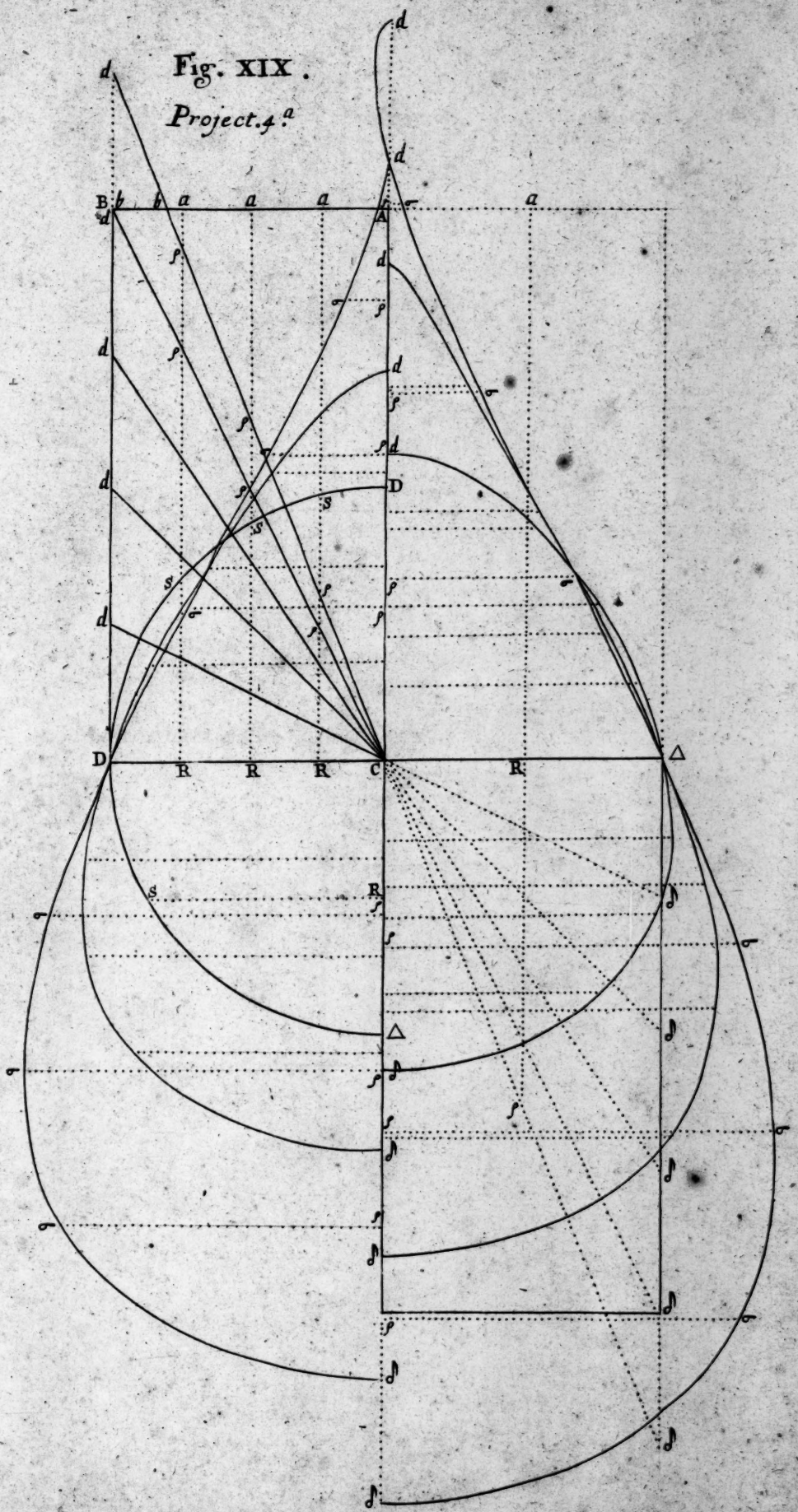


Fig. xx.
Project. 5.ª

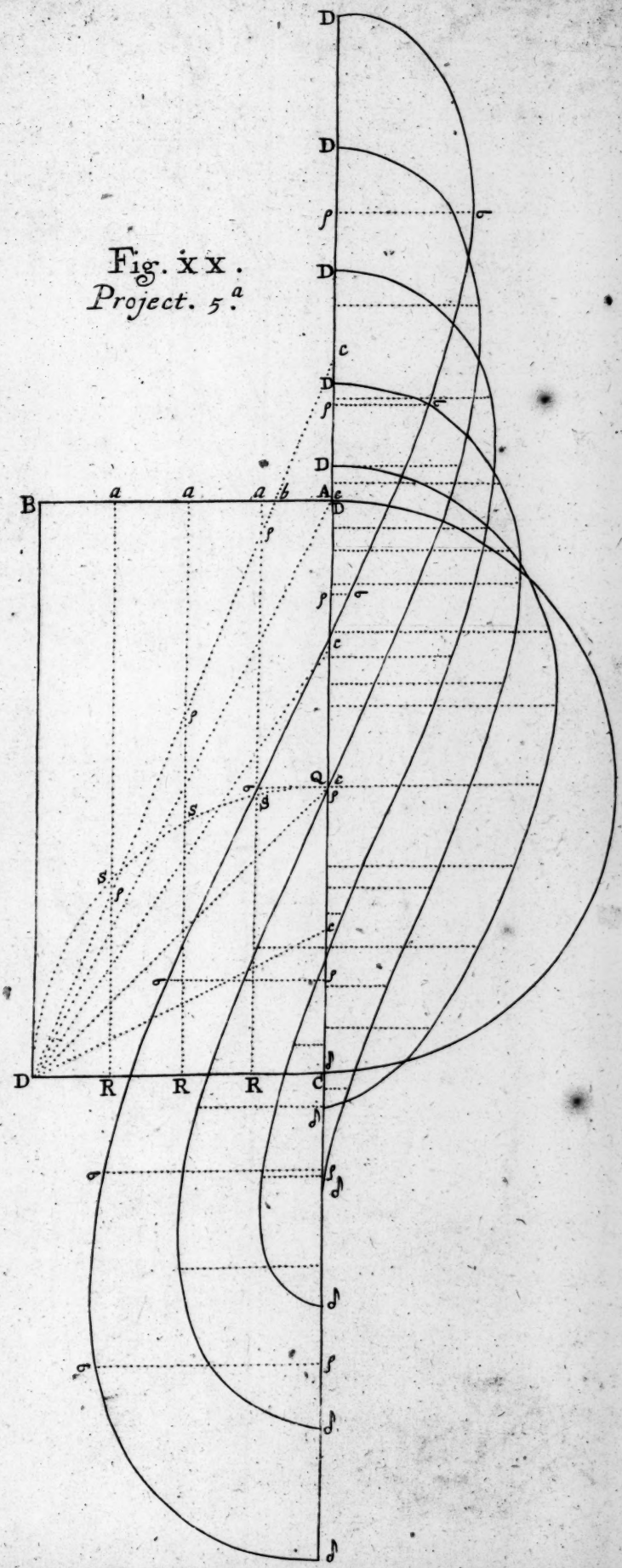


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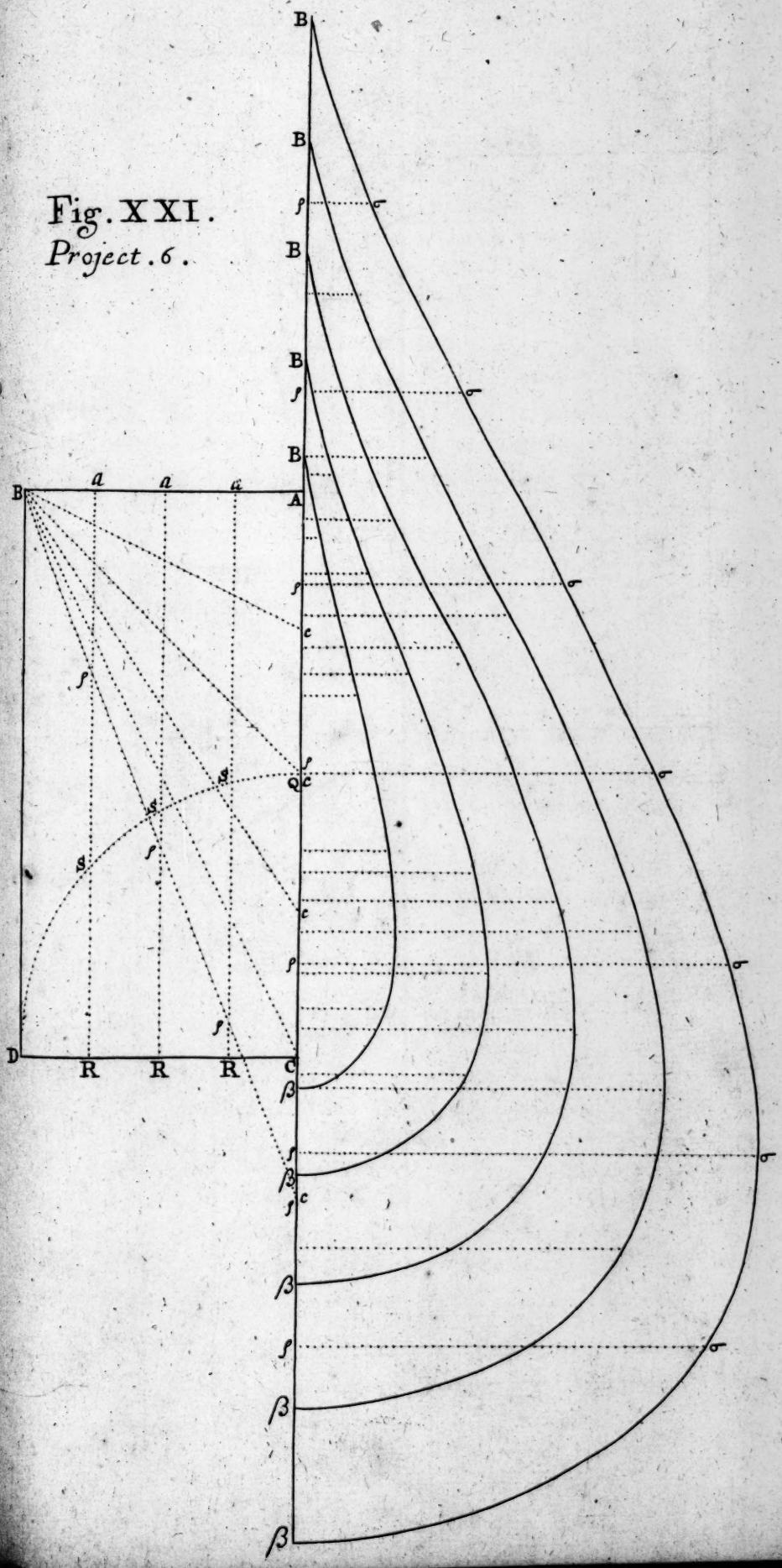
Fig. XXI.
Project. 6.



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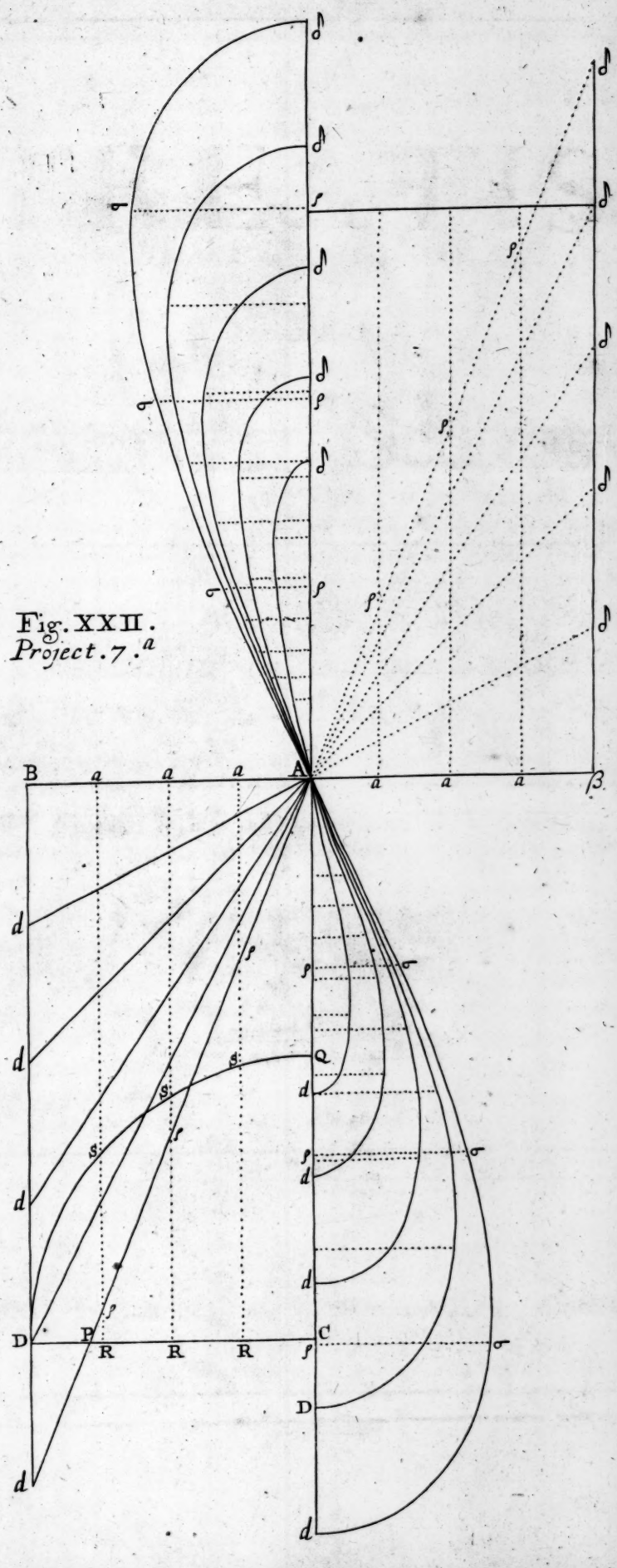
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